

Math308, Quiz 4, 02/14/14

First Name:

Last Name:

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Given the following system of equations:

$$\begin{cases} x_1 & - & x_3 = & -2, \\ 3x_1 & + & x_2 & + & x_3 = & 8, \\ -x_1 & + & x_2 & + & 2x_3 = & 7. \end{cases} \quad (1)$$

Problem 1. 20%. Compute the determinant of the matrix from the system (1).

Problem 2. 80%. Solve the system (1).

Solutions

Problem 1.

$$\det(A) = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 1 \\ -1 & 1 & 2 \end{vmatrix} = -3.$$

Problem 2. The augmented matrix is:

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 3 & 1 & 1 & 8 \\ -1 & 1 & 2 & 7 \end{array} \right).$$

Now, multiply the first row to -3 and add it to the second row; add the first and the third rows:

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 4 & 14 \\ 0 & 1 & 1 & 5 \end{array} \right).$$

Subtract the last row from the second row:

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 3 & 9 \end{array} \right).$$

And the linear system becomes into:

$$\begin{cases} x_1 - x_3 = -2, \\ x_2 + 4x_3 = 14, \\ 3x_3 = 9. \end{cases}$$

Which is easily solved by back substitution:

$$\begin{cases} x_1 = 1, \\ x_2 = 2, \\ x_3 = 3. \end{cases}$$