Math308, Quiz 3, 02/07/14

First Name: .... **Last Name: ....**

Grade: ....

Show all work!

Consider the following initial value problem:

$$(2x - y)dx + (2y - x)dy = 0, \quad y(1) = 0. \tag{1}$$

Problem 1. 20%. Without solving the problem, show that (1) is exact.

Problem 2. 80%. Solve the problem (1).

Solutions

Problem 1. Let us denote: $M(x, y) = 2x - y$ and $N(x, y) = 2y - x$. We check the condition of Theorem 2.6.1:

$$M_y(x, y) = -1, \quad N_x(x, y) = -1.$$

Therefore, given equation is exact.

Problem 2. Since (1) is exact, thus there is a $\psi(x, y)$ such that

$$\psi_x(x, y) = M(x, y) = 2x - y,$$

$$\psi_y(x, y) = N(x, y) = 2y - x.$$

Now, we integrate the first relation and obtain:

$$\psi(x, y) = x^2 - xy + h(y).$$

The next step is to differentiate the last relation with respect to y and compare terms with the definition of $\psi_y(x, y)$:

$$\psi_y(x, y) = -x + h'(y) \qquad = \qquad 2y - x.$$

We see that it must be $h'(y) = 2y$, which gives us $h(y) = y^2 + C$. Therefore, the solution of (1) is the following expression:

$$\psi(x, y) = C, \qquad \text{or } x^2 - xy + y^2 = C.$$

Now, let us apply the initial condition to find the constant C:

$$y(1) = 0 \qquad \Rightarrow \quad 1^2 - 1 \cdot 0 + 0^2 = C \quad \Rightarrow \quad C = 1.$$

And we finally get the solution for the given initial value problem in implicit form:

$$x^2 - xy + y^2 = 1.$$