Math308, Quiz 11, 04/25/14

First Name: .... **Last Name: ....**

Grade: ....

Show all work!

Problem 1. 80%. Find the general solution of the system:

$$\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} \mathbf{x}. \tag{1}$$

Problem 2. 20%. Draw the phase portrait of the above system.

Solutions

Problem 1. We seek solutions of equation (1) of the form $\mathbf{x} = \xi e^{rt}$, where ξ is a vector and r is a scalar. Insert this form into the equation and get the following eigenvalue problem: Find an eigenvector ξ and eigenvalue r such that

$$\begin{pmatrix} -2-r & 1 \\ -5 & 4-r \end{pmatrix} \xi = 0. \tag{2}$$

The eigenvalues are found by setting the determinant of the last matrix to zero:

$$\begin{vmatrix} -2-r & 1 \\ -5 & 4-r \end{vmatrix} = (-2-r)(4-r)+5 = r^2-2r-3 = 0,$$

which gives $r_1 = -1, r_2 = 3$.

Corresponding eigenvectors can be found as follows:

1. $r_1 = -1$

$$\begin{pmatrix} -2-(-1) & 1 \\ -5 & 4-(-1) \end{pmatrix} \xi = 0, \quad \Rightarrow \begin{pmatrix} -1 & 1 \\ -5 & 5 \end{pmatrix} \xi = 0.$$

or

$$-\xi_1 + \xi_2 = 0, \quad \Rightarrow \xi^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

2. $r_2 = 3$

$$\begin{pmatrix} -2-(3) & 1 \\ -5 & 4-(3) \end{pmatrix} \xi = 0, \quad \Rightarrow \begin{pmatrix} -5 & 1 \\ -5 & 1 \end{pmatrix} \xi = 0.$$

or

$$-5\xi_1 + \xi_2 = 0, \quad \Rightarrow \xi^{(2)} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}.$$

The general solution of (1) then is

$$\mathbf{x}(t) = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{3t}.$$

Problem 2.